

# Pricing the Extreme Mortality Bonds Based on the Double Exponential Jump Diffusion Model

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## ABSTRACT

Extreme mortality bonds (EMBs), which can transfer the extreme mortality risks confronted by life insurance companies into the capital market, refer to the bonds whose nominal values or coupons are associated with mortality index. This paper first provides the expected value of mortality index based on the double exponential jump diffusion (DEJD) model under the risk-neutral measure; then derives the pricing models of the EMBs with principal reimbursement non-cumulative and cumulative threshold respectively; finally simulates the bond prices and conducts a parameter sensitivity analysis. This paper finds that the jump and direction characteristics of mortality index have significant impacts on the accuracy of the EMB pricing.

## 1. Introduction

Extreme mortality risk is derived from the situation that the actual mortality rate confronted by life insurance companies is higher than predicted when underwriting the policies. Traditional measures of dealing with the extreme mortality risk include using economic capital to absorb actual losses, selling policies to spread the risk, raising policy rates to transfer costs, and using reinsurance to mitigate risk. However, all these four traditional measures have their own limitations. Therefore, life insurance securitization has been utilized to transfer the extreme mortality risk to the capital market. In 2003, Swiss Reinsurance initiated to apply the method of life insurance securitization to successfully issue the EMBs whose underlying

assets are life insurance policies. Since then, life insurance companies and the academia have been bringing out continuous innovations on the EMBs.

Researches on the EMBs are mainly concentrated in the following three aspects: first, in terms of operational mechanism, Cowley and Cummins (2005), Blake et al. (2006a; 2006b; 2006c), Cairns et al. (2006) and Chen and Cox (2009) study the Vita series EMBs issued by Swiss Reinsurance; Bauer and Kramer (2007) analyze the Tartan EMBs issued by Scottish Reinsurance; second, in terms of mortality index, Dhal et al. (2004), Dowd et al (2006), Cox and Lin (2008), and Deng et al. (2012) explore the jump characteristic of mortality index; third, in terms of pricing model, scholars focus on the imperfect market

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pricing, which includes Wang transform model (Wang, 2000; 2002), Sharp Ratio model (Milevsky et al., 2005), and LFC pricing model (Lane, 2000; Chen and Cummins, 2010).

Although the existing researches on the design and pricing method of mortality index have achieved certain progress, the accuracy of mortality prediction and the rationality of the triggering mechanism still need to be improved. Therefore, based on the double exponential jump diffusion (DEJD) theory proposed by Kou and Wang (2003), employing the research method of Deng et al. (2012), this paper first provides the expected value of mortality index based on the DEJD distribution under the risk-neutral measure, then derives the specific pricing analytic expressions of the EMBs with principal reimbursement non-cumulative and cumulative threshold respectively, and finally simulates the bond prices and conducts the parameters sensitivity analysis.

This paper contributes in the following three aspects: first, based on the assumption that mortality index follows the DEJD process, this paper sufficiently features the direction and frequency of mortality jump, improving the accuracy of the EMB pricing; second, this paper illustrates a much more explicit expression for pricing the EMBs. The results of bond price simulation shows that the EMBs with principal reimbursement non-cumulative threshold are less risky and thus more attractive to investors; third, the parameter sensitivity analysis suggests that the specification of jump direction and frequency influences bond prices significantly, indicating that the description of jump characteristics plays a vital role in the accuracy of the EMB pricing.

## 2. Mortality Index Following DEJD Distribution

The pricing of the EMBs is based on the characterization of mortality index. Different mortality movements result in different bond prices. Considering the changes of mortality rate may not be continuous in reality, which means the standard Brownian motion cannot sufficiently describe the movement of mortality rate, and based on the DEJD theory proposed by Kou and Wang (2003; 2004), this paper argues that the scope of mortality jump follows the DEJD distribution instead of the normal distribution. Utilizing the DEJD distribution to describe the movement of mortality has two obvious advantages. On one hand, it conveniently depicts the mortality jump by simply using different parameters under the same exponential distribution; on the other hand, it effectively characterizes the asymmetry and exponential property of actual mortality jump.

Meanwhile, this paper constructs the mortality index by adopting the risk-neutral pricing measure which is widely

used to price financial derivatives. Based on the risk neutral pricing theory, we can choose a specific risk neutral measure, denoted as  $Q^*$ . Under it, the expected discount for all marketable assets is martingale, and we can obtain the fair price of any security. By the maximum likelihood estimate method, it can implement the risk neutral adjustment to each of the parameters in the mortality time series model, and finally reach the market value of the securities under the risk neutral hypothesis.

Following the research of Deng et al. (2012) and under Lee-Carter (1992) framework,  $\mu_{x,t}$  denotes the mortality rate of the group whose age is  $x$  at time  $t$ . The mortality rate can be expressed by the age-specific parameters  $a_x$ ,  $b_x$  and the mortality time series  $k_t$ :

$$\ln(\mu_{x,t}) = a_x + b_x k_t + e_{x,t} \tag{1}$$

By exponentiating both sides of Equation 1, we can get:

$$\mu_{x,t} = \exp(a_x + b_x k_t + e_{x,t}) \tag{2}$$

To capture the jump characteristic of the mortality time series,  $k_t$  should satisfy the following equation:

$$\frac{dk_t}{k_t} = \alpha dt + \sigma dW(t) + d\left(\sum_{i=1}^{N(t)} (V_i - 1)\right) \tag{3}$$

$N(t)$  is the Poisson process with parameter  $\lambda$ , which represents the jumping frequency.  $V^* = \ln(V^*)$  follows the double exponential distribution:

$$f_{V^*}(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y \geq 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y < 0\}} \tag{4}$$

Where  $p \geq 0$ ,  $q \geq 0$ ,  $p + q = 1$ ,  $\lambda > 0$ ,  $\eta_1, \eta_2 > 0$ . This distribution specifies the upward and downward directions of mortality jump. When  $y \geq 0$ , it describes the sudden surge of mortality rate caused by extreme events. When  $y \leq 0$ , namely the downward jump, it suggests the reduction of mortality rate as a result of the economic development, the increase of living standards, and the improvement of medical and health conditions.

Under the risk-neutral measure, the mortality rate time series satisfies:

$$\frac{dk_t}{k_t} = (\mu^* - \lambda^* \xi^*) dt + \sigma^* dW^*(t) + d\left(\sum_{i=1}^{N^*(t)} (V_i^* - 1)\right) \tag{5}$$

Utilizing *Itô lemma* to solve the above differential equation, we can get:

$$k_t = k_0 + (\mu^* - \frac{1}{2}\sigma^{*2} - \lambda^* \xi^*)t + \sigma^* W_t^* + \sum_{i=1}^{N^*(t)} Y_i^* \tag{6}$$

Therefore, the expected value of overall mortality index is:

$$E^*(\mu_{x,t}) = \exp(a_x) \times E^*[\exp(b_x k_t)]$$

$$= \exp\left(a_x + b_x k_0 + b_x t\left(\alpha^* - \frac{1}{2}\sigma^{*2} - \lambda^* \gamma^*\right) + \frac{1}{2}\sigma^{*2} b_x^2 t + \lambda^* \left(\frac{p^* \eta_1^*}{\eta_1^* - b_x} + \frac{q^* \eta_2^*}{\eta_2^* + b_x} - 1\right) t\right)$$

(7)

Furthermore, the life table can be divided into  $x$  age groups, and each of them is allocated with a weight of  $W_x$ . Then the expectation of the overall mortality rate is:

$$E^*(\mu_t) = \sum_x W_x \times \left\{ \exp(a_x) \times E^*[\exp(b_x k_t)] \right\}$$

$$= \sum_x W_x \times \left\{ \exp\left(a_x + b_x k_0 + b_x t\left(\alpha^* - \frac{1}{2}\sigma^{*2} - \lambda^* \gamma^*\right) + \frac{1}{2}\sigma^{*2} b_x^2 t + \lambda^* \left(\frac{p^* \eta_1^*}{\eta_1^* - b_x} + \frac{q^* \eta_2^*}{\eta_2^* + b_x} - 1\right) t\right) \right\}$$

(8)

### 3. The Pricing Models of Extreme Mortality Bonds based on DEJD Model

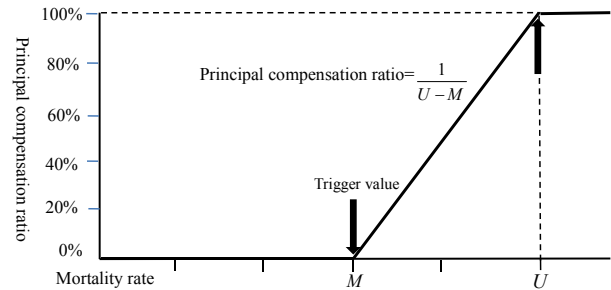
According to whether the current principal compensation payment sets the accumulation of past reimbursement ratios as the trigger condition, the EMBs can be divided into two types of principal reimbursement with non-cumulative threshold bonds and cumulative threshold bonds.

#### 3.1 Pricing Model of Principal Reimbursement with Non-cumulative Threshold EMBs

The trigger condition of the EMBs with principal reimbursement non-cumulative threshold is solely based on the predetermined mortality rate benchmark, i.e. the annual mortality rate benchmark  $k_0$ . SPV obeys the following principles when compensating life insurance companies: (1) if at the time of  $t$ , the mortality index  $k_t$ , surpasses the lower bound of the annual mortality rate benchmark ( $M$ ), SPV will start to reimburse the life insurance companies using the principal raised from the EMBs investors; (2) if the level of mortality rate reaches or even exceeds the upper bound of annual mortality rate benchmark ( $U$ ), SPV will compensate life insurance companies with no more than the entire principal raised by issuing the EMBs; (3) if the mortality level is between the two bounds, SPV will utilize the linear interpolation method to evaluate the compensation ratio from life insurance companies.

From the perspective of a life insurance company, it in fact obtains a series of multi-period call options as the return for the reinsurance premiums it has paid to the SPV. As for bond investors, if trigger events do not happen during the specified period, they will acquire the agreed interest and principal; if extreme events do occur, the bond buyers will lose some or all of the principals. The Vita I EMB of Swiss Reinsurance is a typical representative for the EMB with principal reimbursement non-cumulative threshold, which means the year by year recalculation of

the annual withdrawal and the compensation ratio for the life insurance companies from the SPV, and no relevance to cumulative compensation ratios in the past.



Resource: Klein R., 2006, Mortality catastrophe bonds as a risk mitigation tool, *Society of Actuaries Newspaper*, (57).

**Figure 1.** The Reimbursement Mechanism of the Non-cumulative Threshold EMBs

Figure 1 shows the reimbursement mechanism of the EMBs with principal reimbursement non-cumulative threshold. The black solid line shows the relationship between the principal compensation ratio and the current mortality rate for each year. Let  $q_t$  be the mortality rate at the time of  $t$ , then the principal compensation ratio  $loss_t$  for SPV to compensate the life insurance company is:

$$loss_t = \frac{\text{Max}[q_t - M, 0] - \text{Max}[q_t - U, 0]}{U - M}$$

$$= \begin{cases} 0, & q_t < M \\ \frac{q_t - M}{U - M}, & M < q_t < U \\ 1, & q_t > U \end{cases}$$

(9)

At the maturity date  $T$ , investors can obtain the remaining principal  $FV$  as:

$$FV = \begin{cases} \text{Par} \left(1 - \int_{t=1}^T loss_t dt\right) & \text{if } \int_{t=1}^T loss_t dt \leq 1 \\ 0 & \text{if } \int_{t=1}^T loss_t dt > 1 \end{cases}$$

(10)

Where  $Par$  is the face value of the bond, and  $r_f$  is the risk-free interest rate. Under the risk neutral measure, at maturity date  $T$ , the pricing formula of the EMBs with principal reimbursement non-cumulative threshold can be written as:

$$P = e^{-r_f T} E^*(FV) = \text{Par} \cdot e^{-r_f T} E^* \left[ 1 - \int_1^T loss_t dt \right]$$

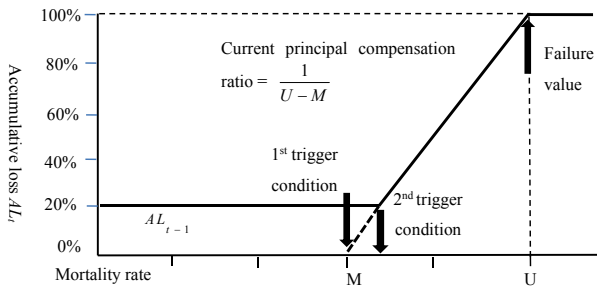
$$= \text{Par} e^{-r_f T} \left[ 1 - E^* \left( \int_1^T loss_t dt \right) \right]$$

$$= \text{Par} \cdot e^{-r_f T} \left[ 1 - \int_1^T \left( \frac{E(\mu_t^*) - M}{U - M} \right) dt \right]$$

(11)

### 3.2 Pricing Model of Principal Reimbursement with Cumulative Threshold EMBs

Due to the fact that mortality rates for successive years generally have sequential correlations, the calculation of mortality rate is usually based on the historical mortality rates. To properly settle the problem of sequential correlations, Scottish Reinsurance issued the Tartan EMBs by designing a double-trigger in 2006. The first trigger condition is the same as that of the non-cumulative threshold EMBs mentioned above, i.e. whether the mortality index surpasses the lower bound of benchmark ( $M$ ); The second trigger condition is whether the principal compensation ratio which is calculated from Formula (9) is greater than the accumulative principal compensation ratio (Accumulated Loss,  $AL_{t-1}$ ), which is the sum of the compensation ratios in last periods.



Resource: Klein R., 2006, Mortality catastrophe bonds as a risk mitigation tool, *Society of Actuaries Newspaper*, (57).

**Figure 2.** The Reimbursement Mechanism of the Cumulative Threshold EMBs

In Figure 2, the black solid line represents the relationship between the accumulated principal compensation ratio of each period and the level of mortality in current period. It can be noted that the accumulated compensation ratio for this period is between the accumulated compensation ratio for the last period and 100%. Only when the current compensation ratio exceeds the cumulative ratio threshold for the last period  $AL_{t-1}$ , will SPV reimburse using the principal. That is to say, at the time of  $t$ , SPV will pay back to life insurance companies at the scale of the accumulated compensation ratio  $AL_t$ :

$$AL_t = \text{Min} \left\{ \text{Max} \left\{ AL_{t-1}, \frac{k_t - M}{U - M} \right\}, 100\% \right\} \tag{12}$$

When the accumulated principal payment ratio hasn't yet reached 100% in the last period, the current principal compensation ratio  $loss_t$  is:

$$loss_t = AL_t - AL_{t-1} = \text{Min} \left\{ \text{Max} \left\{ AL_{t-1}, \frac{q_t - M}{U - M} \right\}, 100\% \right\} - AL_{t-1} = \begin{cases} 0, & \frac{q_t - M}{U - M} < AL_{t-1} \\ \frac{q_t - M}{U - M} - AL_{t-1}, & AL_{t-1} < \frac{q_t - M}{U - M} \leq 100\% \\ 1 - AL_{t-1}, & \frac{q_t - M}{U - M} > 100\% \end{cases} \tag{13}$$

When the bond matures at  $T$ , SPV will pay the principal back to all the bond investors at the scale of  $P_T$ :

$$P_T = 1 - AL_T \tag{14}$$

Therefore the pricing formula of the EMBs with principal reimbursement cumulative threshold is:

$$P = e^{-rT} E^* (FV) = e^{-rT} E^* [1 - (AL_t - AL_{t-1})] = e^{-rT} [1 - E^* (AL_t - AL_{t-1})] = Par \cdot e^{-rT} \left[ 1 - \int_1^T \left( \frac{E^*(\mu_t) - M}{U - M} - AL_{t-1} \right) dt \right] \tag{15}$$

In conclusion, the biggest difference between the two types of the EMBs lies in whether the principal compensation ratio over each mortality assessment period (usually a year) sets the accumulated compensation ratio of last period as the threshold of its lower bound. For the EMBs with principal reimbursement non-cumulative threshold, principal compensation ratios in each year are not related. Once the mortality rate exceeds the lower bound of benchmark ( $M$ ), the principal compensation ratios will increase  $1/(U - M)\%$  with every 1% increase in mortality index. At the same time, if the sum of independent principal reimbursement ratios accumulates to surpass 100% before maturity, SPV will not pay for the bond investors.

However, for the EMBs with principal reimbursement cumulative threshold, not only will the current level of mortality exceed the lower bound of benchmark ( $M$ ), but the proportion of the principal reimbursement ratio should surpass the accumulated proportion in last period  $AL_{t-1}$  as well. Only under this circumstance, SPV will compensate to life insurance companies. In other words, only when the extreme mortality rate is large enough in the second year and meets the double trigger conditions, will the bond investors suffer principal losses, which is more secured to the interests of bond investors. The pricing models of these two types of the EMBs are derived from their corresponding reimbursement mechanisms.

## 4. Numerical Analysis

### 4.1 The Estimation of Extreme Mortality Bond Prices

Assuming that one extreme mortality bond has a maturity of 3 years ( $T = 3$ ) and with a face value of 1 billion RMB ( $Par = 1000000000$ ). This bond was issued at the end of 2013 and will expire at the end of 2016. Risk-free interest rate, represented by the one-year Shibor interest rate, is 0.044. The Benchmark of mortality rate  $\mu_0$  in 2013 is 0.00743. The lower bound of mortality index ( $M$ ) is 1.1 times of the mortality rate in the base year, and the upper bound ( $U$ ) is 1.2 times of the mortality rate in the base year. This paper assumes  $AL = 0.05$  in the pricing formula of the EMBs with principal reimbursement cumulative threshold. Table 1 shows the parameters of annual mortality index.  $W_x$  is a weight associated with the age category.

**Table 1.** The Parameters of Annual Mortality Index

Age Scope	$W_x$	$a_x$	$b_x$
<1	0.013818	-3.4087	0.1455
1-4	0.055317	-6.2254	0.1960
5-14	0.145565	-7.1976	0.1942
15-24	0.138646	-6.2957	0.0994
25-34	0.135573	-5.9923	0.1044
35-44	0.162613	-5.4819	0.0855
45-54	0.134834	-4.7799	0.0608
55-64	0.087247	-4.0137	0.0468
65-74	0.066037	-3.2347	0.0426
75-84	0.044842	-2.4196	0.0409
>85	0.015508	-1.6119	0.0290

Table 2 shows the estimated results of the parameters involved in the pricing model (Deng et al., 2012).

**Table 2.** The Estimates of Parameters

$k_0$	$\alpha^*$	$\sigma^*$	$p^*$	$\gamma^*$	$\eta_1^*$	$q^*$	$\eta_2^*$	$\lambda^*$
-10.302	-0.2	0.31	0.035	-1.25	0.89	0.065	0.93	0.029

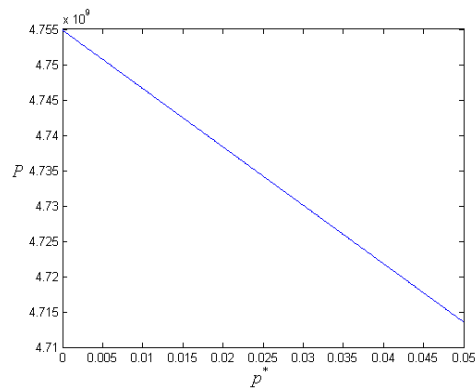
Based on the above pricing models of the EMBs, the bond prices of the two types can be calculated and summarized in Table 3. Comparing the estimated prices of the two EMBs, we can find that the price of the EMB with non-cumulative threshold is lower than that of the EMBs with cumulative threshold, which indicates that the EMBs with non-cumulative threshold are more risky than the cumulative ones, and demand more risk premiums.

**Table 3.** The Estimated Results of the EMB Prices

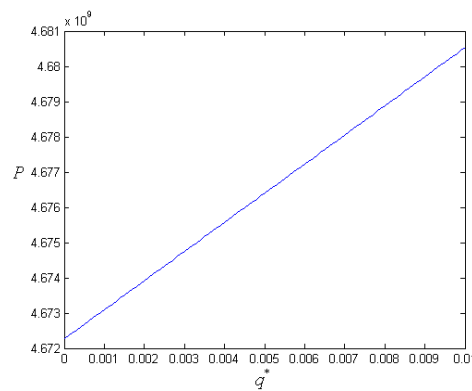
EMB Type	Principal Non-Cumulative	Principal Cumulative
EMB Price	$4.7260 \times 10^9$	$4.9087 \times 10^9$

### 4.2 Parameter Sensitivity Analysis

Figures 1 to 6 plot the relationship between the prices of the EMBs and the main parameters in the pricing models respectively. As in Figure 1 and 2, the parameters  $p^*$  and  $q^*$  which describe the directions of mortality jump have significant impacts on the EMB prices. The upward jump parameter  $p^*$  is negatively correlated with the bond prices; however, the downward jump parameter  $q^*$  is positively correlated with the bond prices. It suggests that when the probability of upward jump rises, the mortality risk in the future will increase, higher compensation bond holders will demand and lower bond prices will be. On the contrary, when the probability of downward jump rises, the mortality risk in the future will decrease, the lower compensation bond holders will demand and higher bond prices will be.



**Figure 1.** The Relation between Price P and  $p^*$



**Figure 2.** The Relation between Price P and  $q^*$

Moreover, Figure 3 shows that the frequency parameter of mortality jump  $\lambda$  also exerts a significant sensitivity impact. When the probability of upward jump is smaller than the downward jump, the frequency parameter  $\lambda$  and the bond prices are linearly positively correlated. This proves that the frequency of mortality jump plays a significant role in the accuracy of pricing the EMBs. Furthermore,

after the distinction of jump directions, both the frequency and direction will influence the mortality risk at the same time, rather than an increase in jump frequency will certainly lead to the increase in the mortality risk when only considering the positive jump. In addition, Figure 4 shows that the bond price is negatively correlated with the parameter  $\alpha^*$ , and has a significant sensitivity.

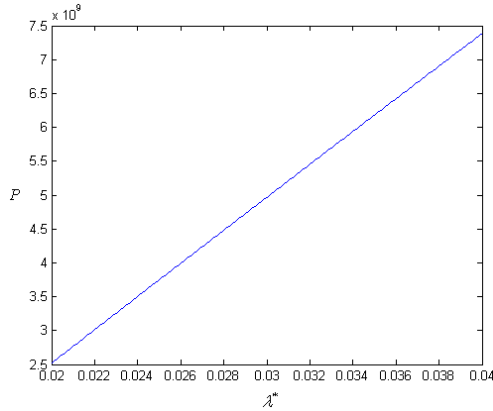


Figure 3. The Relation between Price P and  $\lambda^*$

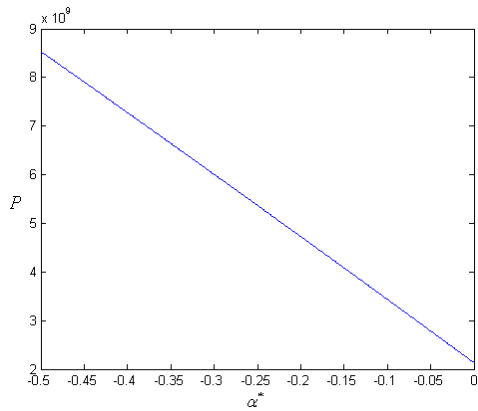


Figure 4. The Relation between Price P and  $\alpha^*$

Finally, Figure 5 and 6 reveal that parameters  $\eta_1$  and  $\eta_2$  which describe the jump scope have a weak correlation and sensitivity relationship with the bond prices. This phenomenon happens after distinguishing the directions of mortality jump.

From the above analysis, it's not hard to notice that when considering the more dedicate descriptions of mortality jump, especially the distinction of the jump directions, can more effectively measure mortality risk and increase the rationality and accuracy of the EMB pricing. Otherwise, only considering upward jump or not distinguishing jump directions is likely to lead to bigger errors in the prediction of mortality index, thus affecting the accuracy of the EMB pricing.

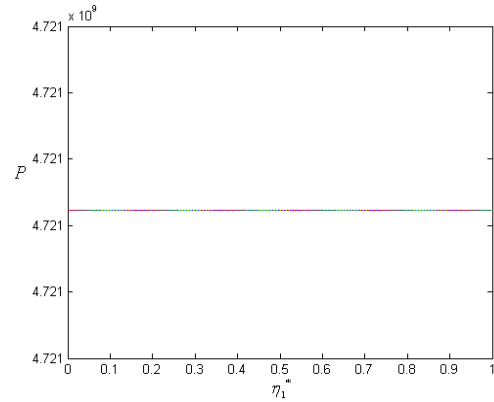


Figure 5. The Relation between Price P and  $\eta_1^*$

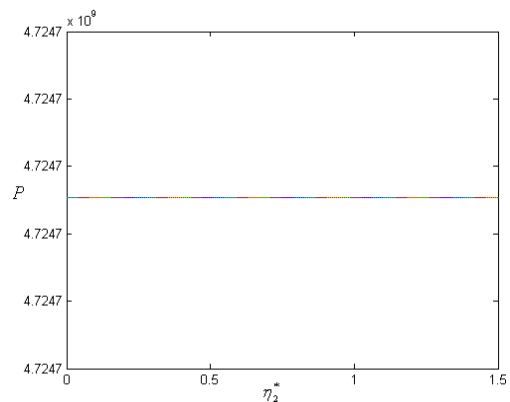


Figure 6. The Relation between Price P and  $\eta_2^*$

## 5. Conclusion

Given the large exposure of extreme mortality risk faced by life insurance companies, how to manage the extreme mortality risk for them has become a hot topic. Since accurately pricing the EMBs is vital to the success of their issuing in the capital market, the focus of this paper is to derive and analyze the EMB pricing model. This paper first introduces a stochastic diffusion model with a double exponential jump diffusion (DEJD) process for mortality time-series. Then, this paper applies the risk neutral pricing theory to derive the pricing models for the EMBs with principal reimbursement non-cumulative and cumulative threshold respectively. The prices of the cumulative threshold EMBs are higher than those of the non-cumulative threshold EMBs, thus more appealing for risk-averse investors. Finally, the results of parameter sensitivity analysis indicate that the mortality jump description, especially the distinction of jump directions, has a significant impact on the rationality of the EMB pricing.

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