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ARTICLE

Boundary Estimation in Annular Two-Phase Flow Using Electrical Impedance Tomography with Particle Swarm Optimization

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ABSTRACT

In this study we consider the boundary estimation of annular two-phase flow in a pipe with the potential distribution on the electrodes mounted on the outer boundary of the pipe, by taking use of electrical impedance tomography (EIT) technique with the numerical solution obtained from an improved boundary distributed source (IBDS) method. The particle swarm optimization (PSO) is used to iteratively seek the boundary configuration. The simulation results showed that PSO and EIT technique with numerical solution obtained from IBDS has been successfully applied to the monitoring of an annular two-phase flow.

1. Introduction

Due to its applications in industry, annular two-phase flow has drawn a lot of attentions in the last decades.[1-3] For annular two-phase flows, the boundary between the liquid and gas phase in the channel is an important aspect for their description. Electrical impedance/resistance tomography (EIT/ERT) is a kind of non-intrusive technique for flow visualization, which has been applied to monitor the multi-phase flow process by many researchers.[2-8] In EIT, a set of electrical currents is injected through a trail of electrodes attached on the boundary of the object and the voltages are measured on the electrodes. Then, with the relationship between the measured voltages and the injected currents the electrical conductivity distribution in the object is reconstructed.

The Finite Element Method (FEM) and Boundary Element Method (BEM) may be the most well-known numerical methods based on mesh for solving EIT forward problems. In contrast, to avoid the disadvantages of numerical methods based on mesh, a new class of numerical methods has been developed only based on a set of nodes without the need for any mesh, called Meshless (or meshfree) Methods (MMs). Many MMs have been proposed and achieved remarkable progress over the last decades.[9-13] In 2013, based on boundary distributed source (BDS)

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method, Kim[14,15] suggested an improved boundary distributed source (IBDS) method, which was considered to be efficient to meet the requirements of high precision for the forward problems of EIT.

Particle swarm optimization (PSO) is a parallel evolutionary computation technique based on the social behavior metaphor, which was proposed by Kennedy and Eberhart.[16, 17] Ijaz et al.[5] employed the PSO for the boundary estimation of an elliptic region using ERT, while this work was based on FEM solution of the forward problem. In 2008, Park et al.[6] reported their work about the monitoring of a radioactive waste separation process by taking use of PSO algorithm based on the analytical solution of EIT problem for concentric cases.

In this study, the IBDS is adopted for the EIT forward problem, and the PSO is employed to seek the boundary configuration of gas phase in an annular flow. The final goal of this study is to estimate the boundary between water phase and gas phase in a pipe with the potential distribution measured on the electrodes mounted on the outer surface of the pipe.

2. Mathematic Model of EIT

The cross section of our mathematic model using in EIT is shown in Figure 1, where σ_b is the conductivity of the water region (background), and u_b the potential distribution of the water region, σ_a the conductivity of the air region and u_a the potential distribution of the air region, respectively. The radius of the pipe is R_1, the radius of the air region is R_2, the relative radius $\rho{=}R_2/R_1$, and the air core located at (x_D, y_D).

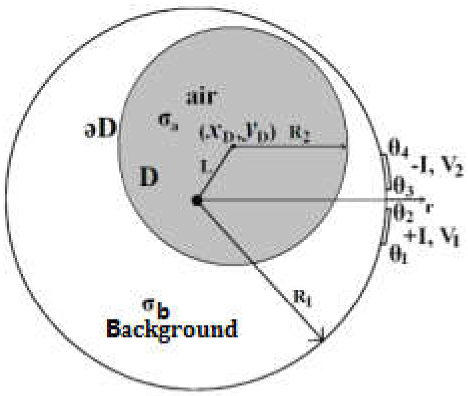

Figure 1. Cross section of the two-phase flow in a pipe with 2 electrodes attached on the boundary

The potential distribution in the subject satisfies the following equation:

$$\nabla \cdot \sigma \nabla u = 0 \tag{1}$$

and the following boundary conditions:

$$\int_{e_l} \sigma \frac{\partial u}{\partial \nu} dS = I_l,\ (x,y) \in e_l,\ l \quad 1,2, \quad ,L \tag{2}$$

$$\sigma \frac{\partial u}{\partial \nu} = 0,\ (x,y) \in \partial\Omega \setminus \bigcup_{l=1}^{L} e_l. \tag{3}$$

$$u + z_l \sigma \frac{\partial u}{\partial \nu} = U_l,\ (x,y) \in e_l,\ l = 1,2,\cdots,L, \tag{4}$$

$$\sum_{l=1}^{L} I_l = 0 \text{ and } \sum_{l=1}^{L} U_l = 0 \tag{5}$$

where ν is the outward normal unit vector on the boundary, $|e_l|$ is the area of the l th electrode, I_l is the current applied to the l th electrode e_l, z_l is the effective contact impedance, U_l is the measured voltage on the l th electrode and L is the number of electrodes.

3. IBDS Method for EIT Forward Problem

Based on the IBDS formulations,[14,15] the potential distribution and its normal derivative on the background $\Omega \setminus \bar{D}$ and on the inclusion D can be expressed as,

$$u_b(p) = \sum_{j=1}^{M} \tilde{G}(p,p_j)\mu_j^b \text{ and } q_b(p) = \sum_{j=1}^{M} \tilde{Q}(p,p_j)\mu_j^b \tag{6}$$

$$u_a(p) = \sum_{j=1}^{M_D} \tilde{G}(p,p_j)\mu_j^a \text{ and } q_a(p) = \sum_{j=1}^{M_D} \tilde{Q}(p,p_j)\mu_j^a \tag{7}$$

Where p is the field point, p_j are the source points, μ_j^b and μ_j^a are the source densities on the boundaries. And the diagonal elements for the Neumann boundary condition are expressed as:

$$\tilde{Q}(p_j,p_j) \cong -\frac{1}{l_j} \sum_{i=1, i\neq j}^{M} \tilde{Q}(p_i,p_j) l_i \tag{8}$$

Thus, the EIT boundary conditions and interfacial conditions can be rewritten as

$$q_b(p_i) = \sum_{j=1}^{M} \tilde{Q}(p_i,p_j)\mu_j^b = 0 \text{ for } p_i \in \partial\Omega_G \tag{9}$$

$$\int_{e_l} \sigma_b q_b(p) dS = I_l \text{ for } l = 1,2,\cdots,L \tag{10}$$

$$u_b(p_i) + z_l \sigma_b q_b(p_i) = U_l \text{ for } p_i \in e_l \text{ and } l = 1,2,\cdots,L \tag{11}$$

$$u_b(p_i) = u_a(p_i) \text{ and } q_b(p_i) = -\kappa q_a(p_i) \text{ for } p_i \in \partial\Omega_D \tag{12}$$

Where $\kappa = \sigma_a / \sigma_b$.

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Integrating equation (12) over an electrode and combining equations (8), (9), and (11) we can have

$$\left[\tilde{G}_{EE}\mu_E^b+\tilde{G}_{EG}\mu_G^b+\tilde{G}_{ED}\mu_D^b\right]+\sigma_b\tilde{D}(z_l)\left[\tilde{Q}_{EE}\mu_E^b+\tilde{Q}_{EG}\mu_G^b+\tilde{Q}_{ED}\mu_D^b\right]=C_U U \tag{13}$$

Where

$$\tilde{G}_{IJ}=\tilde{G}(p_i,p_j)\ \text{ for }\ p_i\in\partial\Omega_I,\ p_j\in\partial\Omega_J\ \text{ and }\ I,J\in\{E,G,D\} \tag{14}$$

$$\tilde{D}(z_l)=\text{diag}[z_l(p_1),z_l(p_2),...z_l(p_{M_E})]\otimes I_{m_E}\in\Re^{M_E\times M_E} \tag{15}$$

$$C_U=\text{eye}(L)\otimes\text{ones}(M_E,1) \tag{16}$$

From the insulation condition and interfacial condition, we have

$$\tilde{Q}_{GE}\mu_E^b+\tilde{Q}_{GG}\mu_G^b+\tilde{Q}_{GD}\mu_D^b=0 \tag{17}$$

$$\tilde{G}_{DE}\mu_E^b+\tilde{G}_{DG}\mu_G^b+\tilde{G}_{DD}\mu_D^b=\tilde{G}_{DD}\mu^a \tag{18}$$

$$\tilde{Q}_{DE}\mu_E^b+\tilde{Q}_{DG}\mu_G^b+\tilde{Q}_{DD}\mu_D^b=-\kappa\tilde{Q}_{DD}\mu^a \tag{19}$$

Integrating the boundary condition and imposing the applied current, we have

$$\frac{1}{|e_l|}\int_{e_l}u_b(p)dS+\frac{z_l}{|e_l|}I_l=U_l\quad\text{for }\ l=1,2,\cdots,L \tag{20}$$

$$\bar{G}_{LE}\mu_E^b+\bar{G}_{LG}\mu_G^b+\bar{G}_{LD}\mu_D^b+D(z_l/|e_l|)\tilde{I}=U=N\beta \tag{21}$$

$$\begin{aligned}U=N\beta&=N(N^TN)^{-1}N^T\left[\bar{G}_{LE}\mu_E^b+\bar{G}_{LG}\mu_G^b+\bar{G}_{LD}\mu_D^b+D(z_l/|e_l|)\tilde{I}\right]\\&=\tilde{N}\left[\bar{G}_{LE}\mu_E^b+\bar{G}_{LG}\mu_G^b+\bar{G}_{LD}\mu_D^b+D(z_l/|e_l|)\tilde{I}\right]\end{aligned} \tag{22}$$

Where $N_U=\left[\text{ones}(1,L-1);-\text{eye}(L-1)\right]$ and $\beta\in\Re^{(L-1)\times 1}$.

Thus the IBDS formulation of the EIT becomes

$$\begin{aligned}&\left[\tilde{G}_{EE}+\sigma_b\tilde{D}(z_l)\tilde{Q}_{EE}\right]\mu_E^b+\left[\tilde{G}_{EG}+\sigma_b\tilde{D}(z_l)\tilde{Q}_{EG}\right]\mu_G^b+\left[\tilde{G}_{ED}+\sigma_b\tilde{D}(z_l)\tilde{Q}_{ED}\right]\mu_D^b\\&=C_U\tilde{N}\left[\bar{G}_{LE}\mu_E^b+\bar{G}_{LG}\mu_G^b+\bar{G}_{LD}\mu_D^b+D(z_l/|e_l|)\tilde{I}\right]\end{aligned} \tag{23}$$

$$\begin{aligned}&\left[\tilde{G}_{EE}+\sigma_b\tilde{D}(z_l)\tilde{Q}_{EE}-C_U\tilde{N}\bar{G}_{LE}\right]\mu_E^b+\left[\tilde{G}_{EG}+\sigma_b\tilde{D}(z_l)\tilde{Q}_{EG}-C_U\tilde{N}\bar{G}_{LG}\right]\mu_G^b\\&+\left[\tilde{G}_{ED}+\sigma_b\tilde{D}(z_l)\tilde{Q}_{ED}-C_U\tilde{N}\bar{G}_{LD}\right]\mu_D^b=C_U\tilde{N}D(z_l/|e_l|)\tilde{I}\end{aligned} \tag{24}$$

We can express equation (24) as

$$V_{EE}\mu_E^b+V_{EG}\mu_G^b+V_{ED}\mu_D^b=C_U\tilde{N}D(z_l/|e_l|)\tilde{I} \tag{25}$$

Where

$$V_{EE}=\tilde{G}_{EE}+\sigma_b\tilde{D}(z_l)\tilde{Q}_{EE}-C_U\tilde{N}\bar{G}_{LE} \tag{26}$$

$$V_{EG}=\tilde{G}_{EG}+\sigma_b\tilde{D}(z_l)\tilde{Q}_{EG}-C_U\tilde{N}\bar{G}_{LG} \tag{27}$$

$$V_{ED}=\tilde{G}_{ED}+\sigma_b\tilde{D}(z_l)\tilde{Q}_{ED}-C_U\tilde{N}\bar{G}_{LD} \tag{28}$$

$$\bar{G}_{LJ}=\frac{s_E}{|e|}\left[I_L\otimes ones(1,m_E)\right]\tilde{G}_{EJ}=A\tilde{G}_{EJ}\quad\text{for }\ J\in\{E,G,D\} \tag{29}$$

Finally, the system equations can be written as

$$\begin{bmatrix}V_{EE}&V_{EG}&V_{ED}&0\\\tilde{Q}_{GE}&\tilde{Q}_{GG}&\tilde{Q}_{GD}&0\\\tilde{G}_{DE}&\tilde{G}_{DG}&\tilde{G}_{DD}&-\tilde{G}_{DD}\\\tilde{Q}_{DE}&\tilde{Q}_{DG}&\tilde{Q}_{DD}&\kappa\tilde{Q}_{DD}\end{bmatrix}\begin{bmatrix}\mu_E^b\\\mu_G^b\\\mu_D^b\\\mu_D^a\end{bmatrix}=\begin{bmatrix}C_U\tilde{N}D(z_l/|e_l|)\tilde{I}\\0\\0\\0\end{bmatrix} \tag{30}$$

When the parameter set X=[x_D, y_D, ρ] is given, the voltages on the electrodes can be calculated from Eqs. (4-7). In the EIT forward problem considered in this paper, the parameters is not known and it should be estimated based on the injected currents and the measured voltage data V_m (l=1,2,…,M). So the problem to be solved is to identify the unknown parameter set [x_D, y_D, ρ] which minimize the difference between the measured voltages V^l induced by the l-th current pattern and the calculated voltage U^l. The object function to be minimized is defined as

$$\psi=\sum_{l=1}^{L}\frac{\left\|V^l-U^l\right\|}{\left\|V^l\right\|} \tag{31}$$

4. PSO Algorithm

The PSO algorithm is initialized with a set of random particles. Each individual in the particle swarm is composed of three K-dimensional vectors. These are the current position $\vec{x}_i$, the previous best position $\vec{p}_i$, and the velocity $\vec{v}_i$. The original process for implementing PSO is as following:[16, 17]

(1) Initialize a set of random particles with random $\vec{x}_i$ and $\vec{v}_i$.

(2) Start loop

(3) For each particle, evaluate the desired optimization fitness function.

(4) Compare particle's fitness evaluation with its $pbest_i$. If current value is better than $pbest_i$, then update $pbest_i$ to the current value, and $\vec{p}_i$ equal to $\vec{x}_i$.

(5) Update $\vec{x}_i$ and $\vec{v}_i$ according to the following equation:

$$\begin{cases} \vec{v}_i \leftarrow c \otimes \vec{v}_i + b_1 \otimes w_1 \otimes (\vec{p}_{Lbest} - \vec{x}_i) + b_2 \otimes w_2 \otimes (\vec{p}_{Gbest} - \vec{x}_i) \\ \vec{x}_i \leftarrow \vec{x}_i + \vec{v}_i \end{cases} \quad (32)$$

(6) If the maximum number of iterations is reached, then exit loop.

In equation (32), w_1 and w_2 are random numbers selected in the range [0,1], and p_{Lbest} denoted the local best solution of the current particle, while p_{Gbest} denoted the global best solution in the whole population. In this paper, we set c=0.6 and b_1=b_2=1.7 as derived by Trelea [18].

5. Numerical Results

In the simulation, suppose 4 electrodes were attached on the boundary. Three current patterns [1, -1, 0, 0], [1, 0, -1, 0] and [1, 0, 0, -1] with the magnitude I_m are used.

In practically, considered the noise of the measured voltages, our data calculated from IBDS consider the measurement error as following:

$$V_{meas} = V(1+\varepsilon_n)$$

Where the measurement error ε_n is assumed to be Gaussian with zero mean. In the PSO algorithm, 10 particles are used, and the maximum number of iterations is 200.

Two numerical examples are simulated in this work, X=[x_D, y_D, ρ]=[0, -0.05, 0.9] and X=[x_D, y_D, ρ]=[-0.3, -0.05, 0.6], the conductivity of water σ_b is set as 1/300 $S\cdot cm^{-1}$, and the conductivity of air σ_a=0. For each case, different percentage of noise are concerned: with 0% noise, 1% noise and 5% noise, respectively.

Table 1 shows the numerical results for the two cases. From the parameters of the numerical examples it can be seen that the PSO technique estimates the location and radius of the air core quite well. The estimated parameters of the two examples are the same as the true value and the object function ψ is smaller than 10^{-5} without noise. For the other cases with noise, the location of the air core is estimated well, and the object function ψ is proportional to the relative noise.

Table 1. Parameters of the numerical examples

True value			Noise	Estimated value			ψ
xD	yD	ρ		xD	yD	ρ	
0	-0.05	0.9	0%	0.0000	-0.0500	0.9000	0.0000
			1%	0.0000	-0.0494	0.9011	0.0442
			5%	0.0000	-0.0485	0.9032	0.2101
-0.3	-0.05	0.6	0%	-0.3000	-0.0500	0.6000	0.0000
			1%	-0.2981	-0.0497	0.6031	0.0490
			5%	-0.2890	-0.0479	0.6132	0.2417

6. Conclusion

In this study we consider the monitoring of an annular two-phase flow using EIT technique. The numerical solution of the forward EIT problem with eccentric cases is derived by using IBDS. The PSO algorithm is employed to estimate the center location and radius of the air core in water-air two-phase flow with the voltage on the electrodes attached on the outer boundary of the pipe, by minimizing a cost functional with the analytical solution. The simulation results showed that PSO and EIT technique has been successfully applied to the monitoring of an annular two-phase flow.

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