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REVIEW

An Optimization Algorithm of Circular Interpolation Aiming at Point-by-Point Comparison Method

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ABSTRACT

Interpolation technology is the core technology of numerical control technology, and the requirements for curve processing in numerical control machine tools are getting higher and higher. According to the current development of numerical control machining, the optimization of the interpolation technology for arcs is proposed,[1] and the improvement is proposed in the original feeding mode which not only improves the calculation speed of the interpolation, but also improves the processing efficiency. In addition, the improved algorithm can reduce the machining error so that the error can be reduced to within the range of 0.5 pulses equivalent.

1. Introduction

At present, computer technology is developing rapidly. Some complicated surface parts can be easily realized by software. The numerical control machine cannot easily realize complicated curve or track processing, most machines use a small straight line segment to force the contour. The smaller the length of the line segment, the closer the feeding path of the tool is to the contour of the part, but it also generates a lot of data. Under normal conditions, the machining accuracy of the machine tool is doubled, and the amount of data generated is several times the accuracy. Conversely, in order to reduce the amount of data, of course, it will facilitate transfer and storage, which will result in reduced processing contour accuracy.[2,3] In response to the above problems, some numerical control systems want to obtain a stable feed rate and perform pre-processing. However, on the common economical numerical control machine, the point-by-point comparison algorithm is used to approximate the line, which may result in a pulse error. An improved scheme for the point-by-point comparison algorithm can reduce the error to within 0.5 pulses,[4] so it is also called the half-step deviation method.

2. The Basic Principle of Half-step Deviation Method

The half-step deviation method is optimized for the feed-by-point comparison method. The point-by-point

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comparison method divides the plane coordinate system into 4 quadrants and moves to the X-axis or Y-axis in the quadrant.[5] As shown in Figure 1 below, the current interpolation point is point P. According to the point-by-point comparison method, the next step is to feed the Y-axis to the point E. The minimum value of this point and the line is L_3. If the X-axis and the Y-axis are fed one step at a time, the point C is reached, and the minimum value of the point C and the straight line is L_2, $L_2 < L_3$; if the current X-axis and Y-axis are set to feed simultaneously (also called diagonal feed), a smaller deviation will be obtained,[6,7] the distance between BC in the figure is 1 pulse, BD + CD = 1, in addition, BD > L_1, CD > L_2; if BD ≥ 0.5, it is known that CD ≤ 0.5, because CD > L_2, it is known that $L_2 < 0.5$; if the next interpolation is performed from the current interpolation point P to reach point C, the error is controlled within the range of 0.5 pulses.

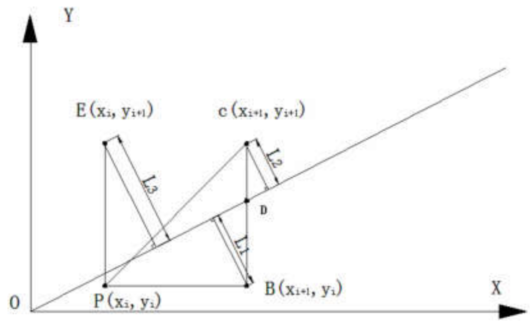

Figure 1. Feeding comparison of interpolation points

3. The Definition of the Deviation Value in Half-step Deviation Method

According to the above inferences, the plane is divided into 8 regions without considering the direction, and used to compare the coordinates of the interpolation end point coordinates, as shown in Figure 2 below.

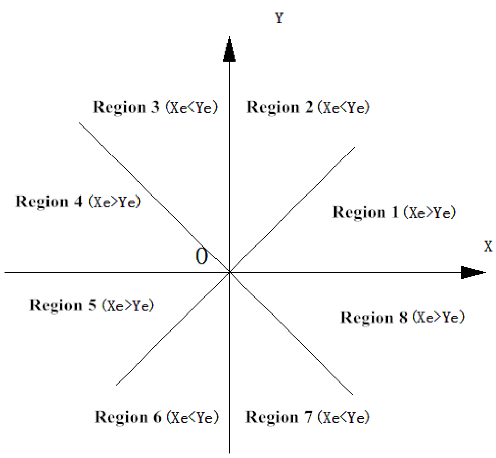

Figure 2. Division of linear interpolation regions by half-step deviation method

The regulations are as shown in Table 1 below:

Table 1

Serial Number	Conditions	The Direction of Feed
1	$X_e \geq Y_e$	Feed towards the one with smaller deviation between the diagonal direction and the X direction.
2	$X_e < Y_e$	Feed towards the one with smaller deviation between the diagonal direction and the Y direction.

The half-step deviation method calculates the one with the smaller deviation between *X/Y* and the diagonal direction before feeding. As the feed direction, for example, there is a point *P* (X_i, Y_i), and the deviation is calculated as follows:

$$F_{ii} = X_e Y_i - X_i Y_e \tag{1}$$

(X_e, Y_e) is the value of the endpoint coordinate.

Starting at point P, if take a step towards X, the new deviation is:

$$F_{i+1,i} = X_e Y_i - (X_i + 1) Y_e \tag{2}$$

If take a step towards Y, the new deviation is:

$$F_{i,i+1} = X_e (Y_i + 1) - X_i Y_e \tag{3}$$

If take a step towards X/Y, the new deviation is:

$$F_{i+1,i+1} = X_e (Y_i + 1) - (X_i + 1) Y_e \tag{4}$$

For the sake of explanation, here is an example of straight line analysis.

Figure 3 is a schematic diagram of the half-step deviation feed. If there is a straight line starting from the starting point, the end point is (X_e, Y_e), and the straight line equation $y=kx(0 < k < 1)$ is in the Region 1. Since $X_e > Y_e$, the feed mode is (△ *x*) or (△ *x*, △ *y*).

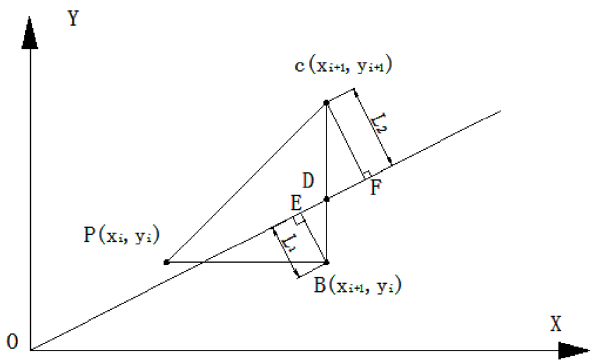

Figure 3. Schematic diagram of the half-step deviation method

The current point is *P* (X_i, Y_i), then there are two pos-

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sibilities for the next move. One is to go one step to the X-axis, to reach B (X_{i+1}, Y_i), and the other is to take the X-axis and Y-axis one step at the same time and reach C (X_{i+1}, Y_{i+1}). According to the first scheme, the distance between the position of the arrival point after interpolation and the straight line is L_1. If the interpolation is performed according to the second scheme, the distance between the position of the point reached after the difference compensation and the straight line is L_2.

As can be seen from Figure 3 above, $\triangle BED$ is similar to $\triangle CFD$.

$BE/CF=BD/CD$

If $BD > Y$, it can be obtained that $L_1 > L_2$, and vice versa. The point at which is located after the interpolation is related to the linear distance and the deviation function. Figure 3 is simplified below to obtain Figure 4.

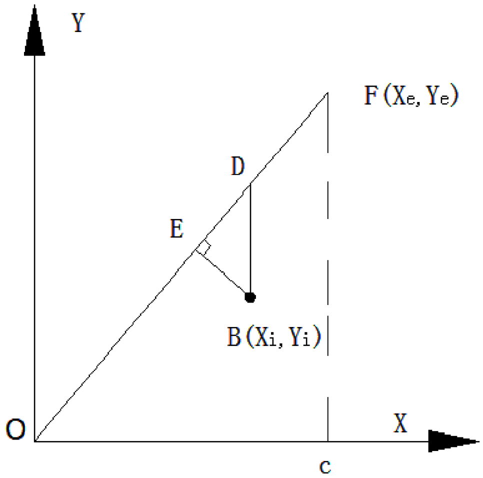

Figure 4. The deviation of the machining points

According to the value of F_{ii}, the distance between point B (X_i, Y_i) and the straight line is obtained, as shown in Figure 4:

$\triangle OFC \backsim \triangle$ BDE, $BE/OC=BD/OF$, and then, it is known that $BE =BD/OF\times OC$. Let $\triangle L$ be the shortest distance from point B to the line, the deviation of B (X_i, Y_i) is:

$F_b=X_eY_i - X_iY_e$

$=X_e[(Y_i + BD) - BD] - X_iY_e$

$=X_eY_D - X_NY_E - X_eBD$

$=-X_eBD$

$=-OC\times BD$

$\therefore \triangle L= |F_B/OF| = |F_B|/OF$ (5)

Where $F_B=F_{ii}$

The deviation value of the $\triangle L$ machining point B from the target straight line is referred to as the deviation.

It can be concluded that, Interpolated from the current point to the next target point (see B, C in Figure 3), The distance between the two target points (B, C) to the straight line (BE, CF) and the BC are linearly divided into two parts (BD, CD) in a proportional relationship ($CF/BE = CD/BD$). (That is to say, in addition to directly comparing the size of $|BE|$ and $|CF|$, the size of $|BD|$ and $|CD|$ can also be compared.)

From the definition of the half-step deviation method, if the interpolation algorithm of the half-step deviation method needs to compare the size of the distance line after the two feeds in the first region, the selection distance is small as the direction. In combination with the above analysis, the following equations (3) and (4) are compared, and the deviation function $|F_{ii} - Y_e|$ after one pulse is emitted in the X-axis direction is compared with the deviation function $|F_{ii} + X_e - Y_e|$ after one pulse is simultaneously emitted to the X-axis and Y-axis.

Set: $f = |F_{ii} + X_e - Y_e| - |F_{ii} - Y_e|$

If $f > 0$, the pulse is simultaneously sent to the X-axis and Y-axis;

If $f \leq 0$, the pulse is sent to the X-axis direction.

3.1 Mathematical Analysis of Half-step Deviation Method

3.1.1 The Circular Feed Rules of Half-step Deviation Method

The interpolation of the arc is more complicated, not only the quadrant problem of the starting point and the end point of the arc, but also the difference between the clockwise arc and the counterclockwise arc. The half-step deviation method circular interpolation is also evolved on the basis of point-by-point comparison method. As shown in Figure 5, the current interpolation point is P. According to the theory of point-by-point comparison method, it can be known:

$$R^2 = X_i^2 + Y_i^2$$

If the point P falls on the arc, the following formula holds:

$$X_i^2 + Y_i^2 = R^2$$

If the point P falls outside the arc, the following formula holds:

$$X_i^2 + Y_i^2 > R^2$$

If the point P falls inside the arc, the following formula holds:

$$X_i^2 + Y_i^2 < R^2$$

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In the point-by-point comparison method, let $X_i^2 + Y_i^2 - R^2$ be the deviation discriminant F_{ii}.

When $F_{ii} \geq 0$, the machining point falls on the arc or outside the arc, and one pulse equivalent is fed to -X at this time;

When $F_{ii} < 0$, the machining point falls within the arc and a pulse equivalent is fed to +Y. Based on the point-by-point comparison method, the following inferences are made:

Let point P be in the first quadrant and the central angle is less than 45°, point P is outside the arc. The next step is to feed the -X direction and reach point C. If let it move (-X, Y) two axes at the same time, reach point B. Let point P coordinate be interpolation coordinate P (X_i, Y_i), point B coordinate be B (X_{i-1}, Y_{i+1}), point C coordinate be C (X_{i-1}, Y_i), $CD + BD = 1$, B, and the distance between B and C is 1 pulse equivalent. As can be seen from the figure, the distance from point C to arc is L_1, and the distance from point B to arc is L_2. The half-step deviation circular interpolation is similar to the half-step deviation linear interpolation. It is necessary to compare the sizes of L_1 and L_2 and select the direction with small deviation.

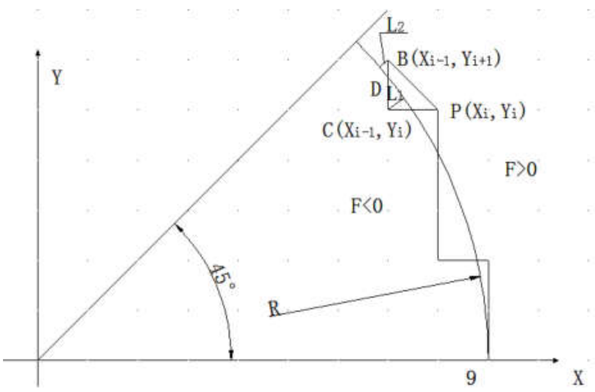

Figure 5. The counterclockwise arc in the first quadrant

According to the point-by-point comparison method region division rule, the half-step deviation circular interpolation divides the plane rectangular coordinate into 8 regions, and the feeding of each region is as shown in Figure 6. In the figure, the big circle is a counterclockwise circular interpolation of the feed route of each region, and the small circle is a clockwise circular interpolation route.

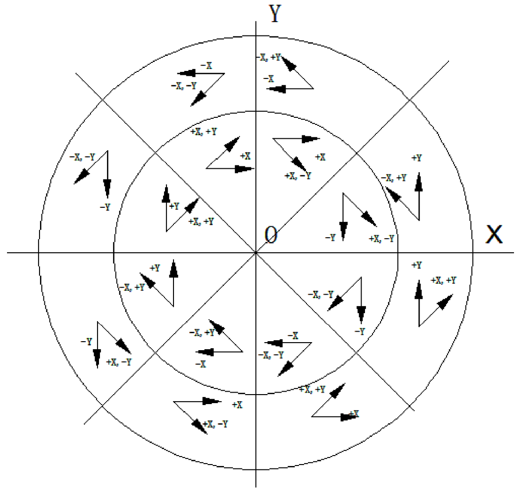

Figure 6. The coordinate feed directions of clockwise and counterclockwise arcs in plane coordinate system

In order to ensure the error of half-step deviation method, the direction of feed is determined according to the following principles:

(1) When $X \geq Y$, one pulse is taken in the positive direction of Y, or one step is taken in the positive direction of -X and +Y.

(2) When $X < Y$, one pulse is taken in the positive direction of X, or one step is taken in the negative direction of -X and +Y.

3.2 The Feed Discriminant of Half-Step Deviation Method

3.2.1 Point-By-Point Comparison Method Interpolation Feed Discriminant and Calculation Formula

In the whole point-by-point comparison method, the addition and subtraction operations and multiplication operations are needed, which directly affects the entire operation speed, if the whole hardware device is added for the dedicated control machine, most of them use its simplified algorithm, called iterative method, and some are called recursive method.

(1) If the first quadrant map is taken as an example, if the deviation value $F_{ii} \geq 0$, one pulse is sent to the positive direction of the X-axis, and the tool is fed one step forward from the current point (X_i, Y_i) to the X-axis to reach the new point (X_{i+1}, Y_i), $X_{i+1} = X_i + 1$, Thus, the deviation function value of the new machining point is:

$$\begin{aligned} F_{i+1,i} &= X_e Y_i - X_{i+1} Y_e \\ &= X_e Y_i - \left(X_i + 1\right) Y_e \\ &= F_{ii} - Y_e \end{aligned} \tag{6}$$

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If the deviation function $F_{ii} < 0$, one feed pulse is sent to the positive direction of the Y-axis, and the tool feeds one step from the current machining point (X_i, Y_i) to the Y direction to reach the point (X_i, Y_{i+1}), where $Y_{i+1}= Y_i+1$, the deviation function value of the point is

$$F_{i,i+1} = X_e\left(Y_i+1\right) - X_iY_e$$
$$= F_{ii} + X_e \tag{7}$$

3.2.2 Half-step Deviation Method Interpolation Feed Discriminant and Calculation Formula

According to the principle of point-by-point comparison method, in order to reduce the infl uence of the algorithm on the operation speed, the half-step deviation arc difference complement algorithm also refers to the recursion method. According to the formula, it can be known that:

(1) When machining from the starting point of the arc, $F_0 = 0$, so use the formula:

$$\begin{aligned} f_0 &= 2F_0 + X_e - 2Y_e \\ &= X_e - 2Y_e \end{aligned} \tag{8}$$

Substituting the value of the actual interpolation arc target, the specific f_0 value is used to determine the feed direction, and its new deviation calculation is determined by the following methods.

(2) If $f_{ii} \geq 0$, the feed X direction is required, and the comparison deviation value of the next point is:

$$\begin{aligned} f_{i+1,i} &= 2F_{i+1,i} + X_e - 2Y_e \\ &= 2\left(F_{ii} - Y_e\right) + X_e - 2Y_e \\ &= F_{ii} + 2X_e - 2Y_e \\ &= f_{ii} - 2Y_e \end{aligned} \tag{9}$$

If $f_{ii} < 0$, $\triangle x$ and $\triangle y$ are fed, then the next deviation is:

$$\begin{aligned} f_{i+1,i+1} &= 2F_{i+1,i+1} + X_e - 2Y_e \\ &= 2\left(F_{ii} + X_e - Y_e\right) + X_e - 2Y_e \\ &= f_{ii} + 2X_e - 2Y_e \end{aligned} \tag{10}$$

According to the above derivation process, a similar formula can be derived when $X_e < Y_e$ or the end point coordinates are in other regions. This directly uses the deviation of the previous step to calculate, which simplifies the calculation formula and improves the calculation efficiency.

4. The Advantages of Half-step Deviation Method

4.1 Circular Interpolation Based on Half-step Deviation Method

A circle with a radius of 10 is processed separately below, and the interpolation results are shown in Figures 7 and 8 below.

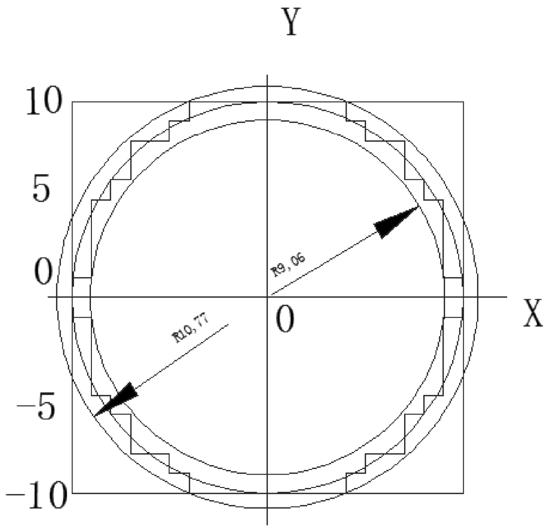

Figure 7. Circular Interpolation Based on Point-by-Point Comparison Method

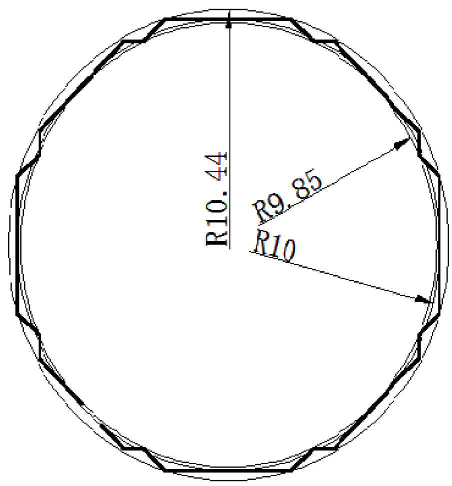

Figure 8. Circular Interpolation Based on Half-step Deviation Method

The interpolation principle of the point-by-point comparison method arc in Figure 7 is as follows: in the first quadrant, starting from the starting point of the arc, counterclockwise interpolation, when the discriminant is less than or equal to 0, go $+Y$, if it is bigger than 0, go to $-X$, and the other three quadrants and so on. and the entire in-

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terpolation route of Fig. 7 can be obtained.

The principle of circular interpolation of the half-step deviation method in Figure 8 is as follows: in the first quadrant, the starting point of the arc starts and counter-clockwise interpolation. Before each interpolation, it is necessary to calculate which of the next two target points is closer to the arc (the distance from the arc is the smallest). After that, the corresponding pulse is sent, and then according to the region where the current point is located, the corresponding interpolation point is selected [in the first region is (-*X*, (-*X, -Y*)], for comparison, by analogy, the second region is selected [-*Y*, (+*X, -Y*)], and the entire interpolation route of Fig. 8 can be obtained according to the inverse circular interpolation shown in Figure 6.

The derivation process has been elaborated above, and the following conclusions can be seen as follows:

(1) After the interpolation by the half-step deviation method, the small circle R9.85 and the large circle R10.44, that is, the deviation contour at the time of interpolation, and the small circle by point-by-point comparison are R9.06 and the large circle R10.77. It is obvious that the accuracy of the half-step deviation method interpolation is high, and the error is within 0.5. According to the discriminating method given above, the interpolation path of the full circle can be derived.

(2) In the point-by-point interpolation comparison method in Figure 8, the whole circle with a processing radius of 10 needs to be calculated 80 times; while the half-step deviation method only needs to calculate 64 times to complete a full circle interpolation, which increases the efficiency by 20%.[8]

5. Conclusion

The point-by-point comparison method and the half-step deviation method are compared with the basic principle, the feed calculation rule, the feed discriminant, etc. The optimized half-step deviation method does not exceed 0.5 pulse equivalents in the interpolation arc error, which is reduced by half compared with the point-by-point comparison method. The half-step deviation method is less than the point-by-point comparison method, and the occupied storage space is small, and the multi-axis linkage and the number of interpolation steps can be reduced.

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